Find the relationship between differential scattering cross-section in the Lab frame and Centre of mass frame:-

$\left(\frac{d\sigma}{d\Omega}\right)_L \rightarrow$ differential scattering cross-section in the Lab frame

$\left(\frac{d\sigma}{d\Omega}\right)_C \rightarrow$ differential scattering cross-section in the centre of mass frame.

Let the number of particles scattered into a given element of solid angle same in both the system,

$$(dN)_L = (dN)_C$$

$$dN = d\sigma \, I$$

$$(I \, d\sigma)_L = (I \, d\sigma)_C$$

$$\cancel{I} \, (d\sigma)_L = (d\sigma)_C \, \cancel{I}$$

$$(d\sigma)_L = (d\sigma)_C$$

$$\left(\frac{d\sigma}{d\Omega} \cdot d\Omega\right)_L = \left(\frac{d\sigma}{d\Omega} \cdot d\Omega\right)_C$$

$$\left(\frac{d\sigma}{d\Omega}\right)_L (d\Omega)_L = \left(\frac{d\sigma}{d\Omega}\right)_C (d\Omega)_C$$

$$\left(\frac{d\sigma}{d\Omega}\right)_L \cancel{2\pi} \sin\theta_{L_1} \, d\theta_{L_1} = \left(\frac{d\sigma}{d\Omega}\right)_C \cancel{2\pi} \sin\theta_{C_1} \, d\theta_{C_1}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_L = \left(\frac{d\sigma}{d\Omega}\right)_C \frac{\sin\theta_{C_1} \, d\theta_{C_1}}{\sin\theta_{L_1} \, d\theta_{L_1}}$$

we know that

$$\tan\theta_{L_1} = \frac{\sin\theta_{C_1}}{\frac{m_1}{m_2} + \cos\theta_{C_1}}$$

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if $m_1 = m_2$

$$\tan\theta_{L_1} = \frac{\sin\theta_{c_1}}{1+\cos\theta_{c_1}}$$

$$\tan\theta_{L_1} = \frac{2\sin\theta_{c_1/2}\cos\theta_{c_1/2}}{\cancel{1}+2\cos^2\frac{\theta_{c_1}}{2}-\cancel{1}}$$

$$\tan\theta_{L_1} = \frac{\cancel{2}\sin\frac{\theta_{c_1}}{2}\cancel{\cos\frac{\theta_{c_1}}{2}}}{\cancel{2}\cos^{\cancel{2}}\frac{\theta_{c_1}}{2}}$$

$$\tan\theta_{L_1} = \tan\frac{\theta_{c_1}}{2}$$

$$\boxed{\theta_{L_1} = \frac{\theta_{c_1}}{2}}$$

$$d\theta_{L_1} = \frac{1}{2}d\theta_{c_1}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_L = \left(\frac{d\sigma}{d\Omega}\right)_c \frac{2\cancel{\sin\frac{\theta_{c_1}}{2}}\cos\frac{\theta_{c_1}}{2}\,\cancel{d\theta_{c_1}}}{\cancel{\sin\frac{\theta_{c_1}}{2}}\times\frac{1}{2}\,\cancel{d\theta_{c_1}}}$$

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_L = \left(\frac{d\sigma}{d\Omega}\right)_c 4\cos\left(\frac{\theta_{c_1}}{2}\right)}$$

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